HENRY ERNEST DUDENEY



536 PUZZLES &

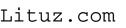


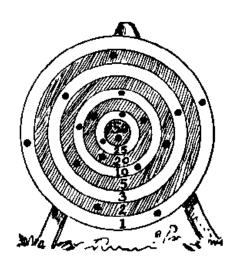
Curious Problems

EDITED AND WITH AN INTRODUCTION BY MARTIN GARDNER, EDITOR OF THE MATHEMATICAL GAMES DEPARTMENT, SCIENTIFIC AMERICAN



Mantesh





536 PUZZLES & CURIOUS PROBLEMS

BY

Henry Ernest Dudeney

EDITED BY MARTIN GARDNER, EDITOR OF
THE MATHEMATICAL GAMES DEPARTMENT,

Scientific American

Charles Scribner's Sons · New York

COPYRIGHT © 1967 CHARLES SCRIBNER'S SONS

This book published simultaneously in the United States of America and in Canada—Copyright under the Berne Convention

All rights reserved No part of this book may be reproduced in any form without the permission of Charles Scribner's Sons

A-9.70[M]

Printed in the United States of America

Library of Congress Catalog Card Number 67-15488

Contents

INTRODUCTION	víi
ARITHMETICAL AND ALGEBRAICAL PROBLEMS	1
Money Puzzles	3
Age Puzzles	11
Clock Puzzles	14
Speed and Distance Puzzles	16
Weight Puzzles	30
Digital Puzzles	32
Skeleton Puzzles	43
Cryptarithm Puzzles	47
Miscellaneous Puzzles	50
GEOMETRICAL PROBLEMS	81
Triangle, Square, and other Polygon Puzzles	83
Circle Puzzles	96
Dividing-the-Plane Puzzles	101
Plane Geometry Puzzles	104

vi	Contents	
	Solid Geometry Puzzles	106
	Dissection Puzzles	112
	Paper Folding Puzzles	127
	Moving Counter Puzzles	130
COM	IBINATORIAL AND TOPOLOGICAL PROBLEMS	139
	Magic Square Puzzles	141
	Magic Star Puzzles	145
	Liquid Pouring Puzzles	149
	Route and Network Puzzles	I 52
	Point Alignment Puzzles	164
	Map Coloring Puzzles	169
	Miscellaneous Combinatorial Puzzles	170
GAM	IE PUZZLES	183
DOM	MINO PUZZLES	189
MAT	CH PUZZLES	199
UNC	LASSIFIED PUZZLES	209
ANS	WERS	223
	INDEX	417

417

Introduction

Henry Ernest Dudeney (the last name is pronounced with a long "u" and a strong accent on the first syllable, as in "scrutiny") was England's greatest maker of puzzles. With respect to mathematical puzzles, especially problems of more than trivial mathematical interest, the quantity and quality of his output surpassed that of any other puzzlist before or since, in or out of England.

Dudeney was born at Mayfield, in Sussex, on April 10, 1857, the son of a local schoolmaster. His father's father, John Dudeney, was well known in Sussex as a shepherd who had taught himself mathematics and astronomy while tending sheep on the downs above Lewes, a town fifty miles south of London. Later he became a schoolmaster in Lewes. Henry Dudeney, himself a self-taught mathematician who never went to college, was understandably proud to be the grandson of this famous shepherd-mathematician.

Dudeney began his puzzle career by contributing short problems to newspapers and magazines. His earliest work, published under the pseudonym of "Sphinx," seems to have been in cooperation with the American puzzlist, Sam Loyd. For a year and a half, in the late 1890's, the two men collaborated on a series of articles in *Tit-Bits*, an English penny weekly. Later, using his own name, Dudeney contributed to a variety of publications including *The Weekly Dispatch, The Queen, Blighty*, and *Cassell's Magazine*. For twenty years his puzzle page, "Perplexities," which he illustrated, ran in *The Strand Magazine*. This was a popular monthly founded and edited by George Newnes, an enthusiastic chess player who had also started and formerly edited *Tit-Bits*.

The Canterbury Puzzles, Dudeney's first book, was published in 1907. It was followed by Amusements in Mathematics (1917), The World's Best Word Puzzles (1925), and Modern Puzzles (1926). Two posthumous collections appeared: Puzzles and Curious Problems (1931) and A Puzzle-Mine (undated). The last book is a mixture of mathematical and word puzzles that Dudeney had

viii Introduction

contributed to *Blighty*. With few exceptions, it repeats puzzles contained in his earlier books. *The World's Best Word Puzzles*, published by the London *Daily News*, contains nothing of mathematical interest.

Dudeney's first two books have, since 1958, been available to American and British readers as paperback reprints. *Modern Puzzles* and *Puzzles and Curious Problems*, in many ways more interesting than the first two books because they contain less familiar puzzles, have long been out of print and are extremely hard to obtain. The present volume includes almost the entire contents of those two books.

Readers familiar with the work of Sam Loyd will notice that many of the same puzzles appear, in different story forms, in the books of Loyd and Dudeney. Although the two men never met in person, they were in frequent correspondence, and they had, Dudeney once said in an interview, an informal agreement to exchange ideas. Who borrowed the most? This cannot be answered with finality until someone makes a careful study of the newspaper and magazine contributions of both men, but it is my guess that most of the borrowing was done by Loyd. Dudeney never hesitated to give credits. He often gives the name or initials of someone who supplied him with a new idea, and there are even occasional references to Loyd himself. But Loyd almost never mentioned anyone. Mrs. Margery Fulleylove, Dudeney's only child, recalls many occasions on which her father fussed and fumed about the extent to which his ideas were being adapted by Loyd and presented in America as the other puzzlist's own. Loyd was a clever and prolific creator of puzzles, especially in his ability to dramatize them as advertising novelties, but when it came to problems of a more mathematically advanced nature, Dudeney was clearly his superior. There are even occasions on record when Loyd turned to Dudeney for help on difficult problems.

Geometrical dissections—cutting a polygon into the smallest number of pieces that can be refitted to make a different type of polygon—was a field in which Dudeney was unusually skillful; the present volume contains many surprising, elegant dissections that Dudeney was the first to obtain. He was also an expert on magic squares and other problems of a combinatorial nature, being the first to explore a variety of unorthodox types of magic squares, such as prime-number squares and squares magic with respect to operations other than addition. (There is an excellent article by Dudeney, on magic squares, in the fourteenth edition of the *Encyclopaedia Britannica*.) In recreational number theory he was the first to apply "digital roots"—the term was probably coined by him—to numerous problems in which their application had not

been previously recognized as relevant. (For a typical example of how digital roots furnish a short cut to an answer otherwise difficult to obtain, see the answer to Problem 131 in this volume.)

Dudeney was tall and handsome, with brown hair and brown eyes, a slightly aquiline nose, and, in his later years, a gray mustache and short chin whiskers. As one would expect, he was a man of many hobbies. "He was naturally fond of, and skilled at games," his wife Alice wrote in a preface to *Puzzles and Curious Problems*, "although he cared comparatively little for cards. He was a good chess player, and a better problemist. As a young man he was fond of billiards, and also played croquet well." In his elderly days he enjoyed bowling every evening on the old bowling green within the Castle Precincts, an area surrounding the ruins of an old castle in Lewes. The Dudeneys owned a two hundred-year-old house in this area, where they were living at the time of Dudeney's death on April 24, 1930. (In Alice Dudeney's preface this date erroneously appears as 1931.)

Mrs. Fulleylove recalls, in a private communication, that her father's croquet lawn, "no matter how it was rolled and fussed over, was always full of natural hazards. Father applied his mathematical and logical skill to the game, with special reference to the surface of our lawn. He would infuriate some of our visitors, who were not familiar with the terrain, by striking a ball in what appeared to be the wrong direction. The ball would go up, down, around the hills and through valleys, then roll gaily through the hoop"

Alice Dudeney speaks of her husband as a "brilliant pianist and organist," adding that, at different times, he was honorary organist of more than one church. He was deeply interested in ancient church music, especially plain song, which he studied intensively and taught to a choir at Woodham Church, Surrey. Mrs. Fulleylove tells me that her father, as a small boy, played the organ every Sunday at a fashionable church in Taunton, Somerset. He was a faithful Anglican throughout his life, attending High Church services, keenly interested in theology, and occasionally writing vigorous tracts in defense of this or that position of the Anglican church.

As a little girl, Mrs. Fulleylove sometimes accompanied her father to his London club for dinner. She remembers one occasion on which she felt very proud and grown-up, hoping the waiter and other guests would notice her sophistication and good manners. To her horror, her father, preoccupied with some geometrical puzzle, began penciling diagrams on the fine damask tablecloth.

In his later life, Mrs. Fulleylove writes, her father lost interest in all

Mantesh

x Introduction

composers except Richard Wagner. "He had complete transpositions for the piano of all Wagner's works, and played them unceasingly—to the great grief of my mother and myself, who preferred the gentler chamber music.

"The house at Littlewick, in Surrey," Mrs. Fulleylove continues, "where we lived from 1899 to 1911, was always filled with weekend guests, mostly publishers, writers, editors, artists, mathematicians, musicians, and freethinkers." One of Dudeney's friends was Cyril Arthur Pearson, founder of the Daily Press and of C. Arthur Pearson, Ltd., a publishing house that brought out Dudeney's Modern Puzzles. Other friends included Newnes and Alfred Harmsworth (later Lord Northcliffe), another prominent newspaper publisher.

"Father provided me, by degrees, with a marvelous collection of puzzle toys, mostly Chinese, in ebony, ivory, and wood...," Mrs. Fulleylove recalls. "He was a huge success at children's parties, entertaining them with feats of legerdemain, charades, and other party games and stunts....

"We had a mongrel terrier that I adored. His name, for some obscure reason, was Chance. One day father fell over the dog's leash and broke his arm. His comment, made without anger, was a quotation: 'Chance is but direction which thou canst not see.'"

In an interview in *The Strand* (April, 1926) Dudeney tells an amusing story about a code message that had appeared in the "agony column" of a London newspaper. A man was asking a girl to meet him but not to let her parents know about it. Dudeney cracked the code, then placed in the column a message to the girl, written in the same cipher, that said: "Do not trust him. He means no good. Well Wisher." This was soon followed by a code message from the girl to "Well Wisher," thanking him for his good advice.

Alice Dudeney, it should be added, was much better known in her time than her husband. She was the author of more than thirty popular, romantic novels. A good photograph of her provides the frontispiece of her 1909 book, A Sense of Scarlet and Other Stories, and her biographical sketch will be found in the British Who Was Who. "A Sussex Novelist at Home," an interview with her that appeared in The Sussex County Magazine (Vol. 1, No. 1, December 1926, pp. 6-9), includes her picture and photographs of the "quaint and curious" Castle Precincts House where she and her husband then lived.

Dudeney himself tried his hand on at least one short story, "Dr. Bernard's Patient," (*The Strand*, Vol. 13, 1897, pp. 50-55). Aside from his puzzle features, he also wrote occasional nonfiction pieces, of which I shall mention only two: "The Antiquity of Modern Inventions" (*The Strand*, Vol. 45, 1913,

хi

p. 389 f) and "The Psychology of Puzzle Crazes" (*The Nineteenth Century*, a New York periodical, Vol. 100, December 1926, p. 868 f).

I have rearranged and reclassified the puzzles that appear in this collection, but only minimally edited the text. British words such as "petrol" have been changed to their American equivalents; long paragraphs have been broken into shorter ones to make for easier reading; and in problems about money American currency has been substituted for British. Some of Dudeney's money problems, so dependent on the relationships between British coins that they cannot be formulated with American currency, have been omitted. In the few cases where duplicate problems, with only trivially different story lines, appeared in the two books I have chosen the version I considered best and left out the other. Titles for problems remain unaltered so that those who may wish to check back to the former appearance of a puzzle can do so easily. The illustrations reproduce the original drawings (some of them done by Mrs. Fulleylove when she was a young girl), enlarged and occasionally retouched to make them clearer.

I have added several footnotes to the puzzles and in the answer section appended a number of comments that are bracketed and initialed. Some of these additions correct errors or point out how an answer has been improved or a problem extended by later puzzle enthusiasts. I hope no one will suppose that these comments reflect in any way on Dudeney's genius. The greatest of mathematicians build on the work of predecessors, and their work in turn is the foundation for the work of later experts. The mathematical-puzzle field is no exception. Dudeney was one of its greatest pioneers, perhaps the greatest, and it is a tribute to him that he was able to invent problems of such depth that decades would pass before others would find ways of improving his answers.

It is Mrs. Fulleylove who is mainly responsible for the book now in the reader's hands. We were in touch first by correspondence; then in 1966, when she took up residence in a New York City suburb, she informed me that she had obtained world reprint rights for *Modern Puzzles* and *Puzzles and Curious Problems*. Would I be interested, she asked, in editing them into a single book? I replied that I would indeed. Enthusiasts of recreational mathematics will rejoice in the appearance of this long inaccessible material, the cream of Dudeney's later years. They will find the book a rich source of unusual problems, many of them leading into fascinating regions that have yet to be fully explored.

xii Introduction

For much of the information in my notes I am indebted to Victor Meally, Dublin County, Ireland. Although he is mentioned often in the notes, there are many places where I followed his excellent and generously given advice without referring to him. I also wish to thank Harry Lindgren, Canberra, Australia; Thomas H. O'Beirne, Glasgow; and C. C. Verbeek, the Hague, for other valuable suggestions.

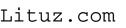
Martin Gardner
HASTINGS-ON-HUDSON, N.Y.

Lituz.com

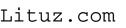
"Amusement is one of the fields of applied mathematics."

W. F. WHITE

A SCRAP BOOK OF ELEMENTARY MATHEMATICS.



Arithmetic & Algebraic Problems



Arithmetic & Algebraic Problems

1. CONCERNING A CHECK

A man went into a bank to cash a check. In handing over the money the cashier, by mistake, gave him dollars for cents and cents for dollars. He pocketed the money without examining it, and spent a nickel on his way home. He then found that he possessed exactly twice the amount of the check. He had no money in his pocket before going to the bank. What was the exact amount of that check?

2. DOLLARS AND CENTS

A man entered a store and spent one-half of the money that was in his pocket. When he came out he found that he had just as many cents as he had dollars when he went in and half as many dollars as he had cents when he went in. How much money did he have on him when he entered?

3. LOOSE CASH

What is the largest sum of money—all in current coins and no silver dollars—that I could have in my pocket without being able to give change for a dollar, half dollar, quarter, dime, or nickel?

4. GENEROUS GIFTS

A generous man set aside a certain sum of money for equal distribution weekly to the needy of his acquaintance. One day he remarked, "If there are five fewer applicants next week, you will each receive two dollars more." Unfortunately, instead of there being fewer there were actually four more persons applying for the gift.

"This means," he pointed out, "that you will each receive one dollar less." How much did each person receive at that last distribution?

4 Arithmetic & Algebraic Problems

5. BUYING BUNS

Buns were being sold at three prices: one for a penny, two for a penny, and three for a penny. Some children (there were as many boys as girls) were given seven pennies to spend on these buns, each child to receive exactly the same value in buns. Assuming that all buns remained whole, how many buns, and of what types, did each child receive?

6. UNREWARDED LABOR

A man persuaded Weary Willie, with some difficulty, to try to work on a job for thirty days at eight dollars a day, on the condition that he would forfeit ten dollars a day for every day that he idled. At the end of the month neither owed the other anything, which entirely convinced Willie of the folly of labor. Can you tell just how many days' work he put in and on how many days he idled?

7. THE PERPLEXED BANKER

A man went into a bank with a thousand dollars, all in dollar bills, and ten bags. He said, "Place this money, please, in the bags in such a way that if I call and ask for a certain number of dollars you can hand me over one or more bags, giving me the exact amount called for without opening any of the bags."

How was it to be done? We are, of course, only concerned with a single application, but he may ask for any exact number of dollars from one to one thousand.

8. A WEIRD GAME

Seven men engaged in play. Whenever a player won a game he doubled the money of each of the other players. That is, he gave each player just as much money as each had in his pocket. They played seven games and, strange to say, each won a game in turn in the order of their names, which began with the letters A, B, C, D, E, F, and G.

When they had finished it was found that each man had exactly \$1.28 in his pocket. How much had each man in his pocket before play?

9. DIGGING A DITCH

Here is a curious question that is more perplexing than it looks at first sight. Abraham, an infirm old man, undertook to dig a ditch for two dollars. He engaged Benjamin, an able-bodied fellow, to assist him and share the money fairly according to their capacities. Abraham could dig as fast as Benjamin could shovel out the dirt, and Benjamin could dig four times as fast as Abraham could do the shoveling.

How should they divide the money? Of course, we must assume their relative abilities for work to be the same in digging or shoveling.

10. NAME THEIR WIVES

A man left a legacy of \$1,000.00 to three relatives and their wives. The wives received together \$396.00. Jane received \$10.00 more than Catherine, and Mary received \$10.00 more than Jane. John Smith was given just as much as his wife, Henry Snooks got half as much again as his wife, and Tom Crowe received twice as much as his wife. What was the Christian name of each man's wife?

11. MARKET TRANSACTIONS

A farmer goes to market and buys a hundred animals at a total cost of \$1,000.00. The price of cows being \$50.00 each, sheep \$10.00 each, and rabbits 50ϕ each, how many of each kind does he buy? Most people will solve this, if they succeed at all, by more or less laborious trial, but there are several direct ways of getting the solution.

12. THE SEVEN APPLEWOMEN

Here is an old puzzle that people are frequently writing to me about. Seven applewomen, possessing respectively 20, 40, 60, 80, 100, 120, and 140



To'liq qismini Shu tugmani bosish orqali sotib oling!